MATH 512 HOMEWORK 4

Due Friday, April 12

Recall that if $F : \lambda^{<\omega} \to \lambda$, $C_F := \{x \mid F"x^{<\omega} \subset x\}$ i.e. the set of all closure points of F.

Problem 1. *Suppose that $\kappa > \omega_1$.*

(1) *Show that $\{x \in \mathcal{P}_\kappa(\lambda) \mid |x| \geq \omega_1\}$ is club in $\mathcal{P}_\kappa(\lambda)$.*
(2) *Use that to show that $\langle C_F \mid F : \lambda^{<\omega} \to \lambda\rangle$ do not generate the club filter.*

Problem 2. *Show that the club filter on $\mathcal{P}_\kappa(\lambda)$ is generated by the sets $\langle C_F \mid F : \lambda^{<\omega} \to \lambda\rangle$ and $\{x \mid \kappa \cap x \in \kappa\}$*

Recall that q is $(M, \mathbb{P})$-generic if for every maximal antichain $A \in M$, $A \cap M$ is predense below q (i.e. every $r \leq q$ is compatible with some $s \in A \cap M$).

Problem 3. *Suppose that $\mathbb{P}$ is a forcing notion and $q \in \mathbb{P}$. Let λ be large enough and $M \prec H_\lambda$ be such that $\mathbb{P} \in M$. Show that the following are equivalent:*

(1) *q is $(M, \mathbb{P})$-generic.*
(2) *$q \Vdash$ "$\dot{G} \cap M$ is a $\mathbb{P}$-generic filter over M".*

Problem 4. *Suppose that $\mathbb{P} * \dot{\mathbb{Q}}$ is a two step iteration and $M \prec H_\lambda$ is a countable structure with $\mathbb{P} * \dot{\mathbb{Q}} \in M$. Suppose that $p \in \mathbb{P}$ and $\dot{q}$ is a $\mathbb{P}$-name for a condition in $\dot{\mathbb{Q}}$, such that*

- *p is $(M, \mathbb{P})$-generic, and*
- *$p \Vdash_{\mathbb{P}}$ "$\dot{q}$ is $(M[\dot{G}_P], \dot{\mathbb{Q}})$-generic".*

Show that $\langle p, \dot{q}\rangle$ is $(M, \mathbb{P} * \dot{\mathbb{Q}})$-generic.
(Here $\dot{G}_P$ is the canonical name for a generic filter for $\mathbb{P}$.)

Problem 5. *Suppose that $\mathbb{P}$ is a forcing notion that does not preserve stationary subsets of ω_1. Show that there is a collection $\langle D_\alpha \mid \alpha < \omega_1\rangle$ of dense subsets of $\mathbb{P}$, such that there is no filter $G \subset \mathbb{P}$ in the ground model that meets them all.*

Remark 1. The above problem shows that we cannot have a forcing axiom for posets that are not stationary set preserving for subsets of ω_1.